1. Title of Submission: On Culturally and Racially Inclusive Mathematics Curriculum.

2. Name of Author: Francis Ahia.

3. Affiliation: Ontario Institute for Studies in Education, University of Toronto, Department of Curriculum, Teaching and Learning / Transitional Year Program.

4. Address of Author: 252 Bloor Street West, Toronto Ontario, M5S 1V6, Canada.

5. E-mail Address: francis.ahia@utoronto.ca

6. Abstract of Paper:

Culturally and racially inclusive mathematics curriculum has become important in the quest for inclusive education for all peoples especially in a multiracial and multicultural society. The mathematical methods used in antiquity by the older civilizations; Babylonians, Egyptians, Indians, Arabs, and their pivotal contributions to the development of mathematics, can be used to address some of the issues of inclusive curriculum. In this paper, I discuss the relevance of some mathematical methods and concepts of the Babylonians and Egyptians to the development of an inclusive mathematics curriculum. In the case of fractions, I show that the Egyptian concept and methods in dealing with fractions, enabled them to raise and answer some questions, which would be difficult to answer with the methods in use at present.

ON CULTURALLY AND RACIALLY INCLUSIVE MATHEMATICS CURRICULUM

ABSTRACT:

Culturally and racially inclusive mathematics curriculum has become important in the quest for inclusive education for all peoples especially in a multiracial and multicultural society. The mathematical methods used in antiquity by the older civilizations; Babylonians, Egyptians, Indians, Arabs, and their pivotal contributions to the development of mathematics, can be used to address some of the issues of inclusive curriculum. In this paper, I discuss the relevance of some mathematical methods and concepts of the Babylonians and Egyptians to the development of an inclusive mathematics curriculum. In the case of fractions, I show that the Egyptian concept and methods in dealing with fractions, enabled them to raise and answer some questions, which would be difficult to answer with the methods in use at present.

INTRODUCTION:

In this paper, I examine ways in which ancient Egyptian method of multiplication, concept and method of dealing with fractions, ancient Babylonian method of solving a system of linear equations, quadratic equations, and method of solving right triangles, would fit into a mathematics curriculum. My goal is to prod mathematics educators to help end the mono-culture of our curriculum and make our classrooms truly multicultural and multiracial by bringing the knowledge of all races and cultures to the classroom.

Harding (1993), defines eurocentrism as, the assumption that Europe functions autonomously from other parts of the World; that Europe is its own origin, final end, and agent; that Europe and people of European descent in the Americas and elsewhere owe nothing to the rest of the world. Analogously the present mathematics curriculum assumes that the origin of mathematics is Europe; that mathematics owes its development to Europe and people of European descent in the Americas and elsewhere but owe nothing to the rest of the World. In short the present mathematics curriculum is eurocentric. But the assumption that mathematics owes its origin and development to Europe is false, for the fact that the present number system which is fundamental to mathematics has its origin in the (Hindu) Indian and Arabic cultures, and owes nothing to Europe in its origin or development. These misrepresentations cannot be corrected by the concept and development of "ethno" mathematics since such an approach leaves eurocentric mathematics setting the standard for real "mathematics" (analogous with Harding 1993), and more importantly does not correct the underlying false assumptions of the present curriculum. The eurocentric curriculum is also a contributing factor to a sizable number of Black students disengagement from the school system (Dei et al 1997). A new curriculum with underlying assumptions acknowledging the important contribution of all cultures and races to the origin and development of mathematics is needed. Such a curriculum is what we define as culturally and racially inclusive curriculum. The debate over the use of such an inclusive curriculum is not new.

Around the 1840s, a Lancelot Wilkinson, a British colonial officer advocated and successfully used such a curriculum (in the context of Indian mathematical and scientific heritage) in his school in India. He also makes a reference to a similar approach, which had succeeded in Ceylon (now Sri Lanka) (Adas 1989). By curriculum we mean the parts that deal with the content of what is taught and the method of teaching it. Teachers have the most control of these parts of the curriculum (Arshad 2001). Such inclusive curriculum brings the important contributions of all cultures and races to a subject to the classroom, and therefore fosters anti-racist education and truly makes the classroom multicultural and multiracial. Such a curriculum is also anti-colonial, since the important contribution of all cultures and peoples to knowledge is taught. The British colonial administration in India dismissed as irrelevant and of no value for study, Indian achievements in all fields, and imposed European education on India (Adas 1989). Above all an inclusive curriculum will enhance mathematics education in so many ways, among which are; the historic approach to mathematics, which will help students to appreciate mathematics as a living subject whose growth is nurtured by all cultures, peoples and generations; students will be introduced to the best methods and concepts from all cultures and peoples. The hope is that an inclusive curriculum will also enhance the learning of mathematics by all students and may help to address the under representation of some minority groups in the mathematical sciences. The approach of consigning equity issues as relevant only in certain subject areas in the social sciences and mainly for minority ethnic students fails to address the major issues of multicultural education. Yet this and the celebration of cultural festivals are the main characteristics of present multicultural education (Arshad 1989). Multicultural education should celebrate not only festivals but also the thoughts and knowledge of all cultures and peoples in all subject areas and for all students.

The paper will draw heavily on the translation of ancient Egyptian and Babylonian texts as presented in the references: Aaboe (1975), Archibald (1941)., Gillings (1982)., Newman (2000)., Robins and Shute (1987).

EGYPTIANS: (Aaboe, Archibald 1941, Gillings 1982, Newman 2000, Robins and Shute 1987)

We will first demonstrate the Egyptian method of multiplication with an example. We will then discuss the method, indicate where it will fit in a mathematics curriculum, and show how it helps to increase the understanding of some of the methods of Arithmetic in present use.

MULTIPLICATION:

Example: Find the product of 37 and 58, that is: Find 37 x 58 = ?

37

→ 1	58
+ 1	58
2	116
+ 2	116
→ 4	232
+ 4	232
8	464
+ 8	464
16	928
+16	928
→ 32	1856

⇨

Sum of numbers that the arrows point to

Left Column:

1 + 4 + 32 = 37

Right Column:

58
232
1856
2146

Product:

37 x 58 = 2146

The sum of the marked numbers (1, 4, 32) in the left column is equal to the multiplicand, 37. The sum of the corresponding numbers (58, 232, 1856) in the right column, 2146, is the product of 37 and 58.

Discussion:

The first row, always has 1 in the left column, and the multiplier in the right column: in the example,1 and 58 respectively. Starting with the first row, each successive row is obtained by doubling the previous row, that is by adding the previous row to itself. The entry in the left column of a row cannot be greater than the multiplicand, which is put on top of the left column as a reminder. The process of successive doubling therefore is stopped if further doubling produces a left column entry greater than the multiplicand. In the example 32 + 32 = 64, which is greater than 37. We therefore stopped at the row with left column entry 32. By trial and error, numbers in the left column are found which add up to the multiplicand: in the example 1, 4, 32 which add up to 37. Then the sum of the corresponding numbers in the right column is the product of the multiplicand and multiplier: in the example, 2146 the sum of 58, 232, 1856 is the product of 37 and 58.

The method implicitly uses two important properties of numbers:

a) Any number can be expressed as the sum of powers of the number 2

b) Multiplication is distributive over addition.

Each of the left column entries is a power of 2, and so a sum of any of the left column entries is a sum of powers of 2. The multiplicand is therefore expressed as a sum of powers of 2. Each of the right column entries is the product of the corresponding right entry and the multiplier, and so a sum of any of the right column entries is a sum of multiples of the multiplier. The product is therefore a sum of multiples of the multiplier, and the sum of the multiples is the multiplicand. In the example:

$37 = 1 + 4 + 32$ but $1 = 2^0,\ 4 = 2^2,\ 32 = 2^5$ and so 37 is expressed as sum of powers of 2.

$2146 = 58 + 232 + 1856$ *but* $58 = 1 \times 58,\ 232 = 4 \times 58,\ 1856 = 32 \times 58.$
$2146 = (1 + 4 + 32)58.$

The Egyptian method of multiplication is therefore, not only ingenuous but also theoretically sound

Place of Method in a Curriculum:

It requires only addition to use the method and one does not need to know the multiplication table to use the method. As such it will fit at the place in a curriculum after addition or could be taught as application of addition. It can also be modified (procedure for expressing the multiplicand as sum of powers of 2) and taught as one of the methods that can be used to express a given number as sum of the powers of 2. The method can also be adapted to construct a multiplication table (see appendix) and so in this modified form can be taught at the beginning of teaching multiplication.

Impact on Present Methods:

The Egyptian method unlike the present method of multiplication emphasis the fact that Multiplication is Addition. It also gives an insight that the Multiplication table as is called can also be called Addition table. To the Egyptians division and multiplication are the 'same' operations and hence used the 'same' procedure for both. The only difference is that in multiplication they used the left column to find the product in the right column whilst in division they used the right column to find the quotient in the left column. To them to find the quotient is to find the number that multiplies the divisor to give the dividend. $1755 \div 27$ means what do you have to multiply by 27 to get 1755. This transforms division to multiplication.

Division:

Example: Divide 1755 by 27, that is Find $1755 \div 27 = ?$

	1755
1	27 ←
+ 1	27
2	54
+ 2	54
4	108
+ 4	108
8	216
+ 8	216
16	432
+ 16	432
32	864
+ 32	864
64	1728 ←

⇨

Sum of Numbers that Arrows point to:

Right Column:
1728
+ 27
1755

Left Column:
a) + 64 = 65

Quotient:
1755 ÷ 27 = 65

The Egyptians approach of seeing both multiplication and division as addition, and their method of explicitly using addition for these two operations would make it easier for students to appreciate the interconnectedness between the three operations and hopefully, help to dispel the notion that some operations are more complicated than others.

because of their 'pictorial' number system (hieroglyphics) were masters at addition. They used addition to carry out the other basic arithmetic operations: subtraction, multiplication and division. They performed multiplication by ingeniously using the fact that (i) any number can be expressed as a sum of powers of 2, (ii) multiplication is distributive over addition.

FRACTIONS:

EGYPTIANS: A part where two halves make a whole, three thirds make a whole, four quarters make a whole and so on.

That is:

$a)\ \frac{1}{2} + \frac{1}{2} = 1$

$b)\ \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$c)\ \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$

Except $\frac{2}{3}$, the Egyptians wrote their fractions as sums of fractions with numerator one.

Example: $\frac{3}{4}$ *as the sum of* $\frac{1}{2}$ *and* $\frac{1}{4}$

The Egyptians might have done this for utility in the sense that if 9 loaves are shared among 10 men, it is not enough to know that each receives $\frac{9}{10}$ of a loaf, but to have a method of sharing. From the Rhind Mathematical Papyrus if 10 men are to share 9 loaves equally, each man according to A'hmose is to get $\frac{2}{3}, \frac{1}{5}, \frac{1}{30} \left(i.e\ \frac{27}{30}\right)$. That is divide each of 7 loaves into 3 equal parts. 21 of $\frac{1}{3}$ loaf are obtained. Take one of the $\frac{1}{3}$ loaves and divide it into 10 equal parts. Divide each of the remaining two loaves into 5 equal parts. The 9 loaves are now divided into

i) 20 of $\frac{1}{3}$ loaf; ii) 10 of $\frac{1}{5}$ loaf; and iii) 10 of $\frac{1}{30}$ loaf.

So each of the 10 men receive 2 of $\frac{1}{3}$ loaf that is $\frac{2}{3}$ loaf, one of $\frac{1}{5}$ loaf, and one of $\frac{1}{30}$ loaf.

So the Egyptians answer the more difficult question how do you divide 9 loaves, **equally** among 10 men. We should therefore also ask how you share 9 loaves equally (that is the same number of fractional parts) among 10 men

BABYLONIAN: (Aaboe 1975, Archibald 1941, Allen 1997)

The legacy of the Babylonian sexagesimal number system is present in the relationships between the units we use for time (hours, minutes, seconds) and angles (degrees, minutes, seconds).

We will look at the methods used by the Babylonians to solve systems of linear equations and quadratic equations. The methods translated into our present terminology are simply elegant. They are also simple enough that they would be excellent in introducing these topics to students.

SYSTEM OF LINEAR EQUATIONS

The Babylonians turn a system of two linear equations in two variables into a linear equation in one variable. The method uses the fact that if A is the average of the two numbers X and Y, then there is a number D such that: A + D = X, and A - D = Y.

Problem 1: *Solve*

$$\begin{aligned} \frac{2}{3}x - \frac{1}{2}y &= 500 \\ x + y &= 1800 \end{aligned}$$

Solution:

$$\text{Select} \quad \tilde{x} = \tilde{y} \quad \text{such that} \quad \tilde{x} + \tilde{y} = 1800$$

$$\text{So} \quad \tilde{x} = 900$$

$$\text{Now make the model}: \quad x = \tilde{x} + d, \qquad y = \tilde{y} - d$$

$$\text{That is}: \quad x = 900 + d, \qquad y = 900 - d$$

$$\text{Substitute in first equation}:$$

$$\frac{2}{3}(900 + d) - \frac{1}{2}(900 - d) = 500$$

$$\text{Solve the above linear equation}: \quad d = 300$$

$$\text{Then}: \quad x = 1200, \quad y = 600$$

GENERAL SYSTEM OF TWO LINEAR EQUATIONS IN TWO VARIABLES:

For a solution of system of equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned}$$

First use the following transformations:

$z_1 = a_{21}x_1, \qquad z_2 = a_{22}x_2$

Then the equation is transformed into :

$$\left(\frac{a_{11}}{a_{21}}\right)z_1 + \left(\frac{a_{12}}{a_{22}}\right)z_2 = b_1$$

$$z_1 + z_2 = b_2$$

The method is then used to solve the above system of equations for z_1 and z_2. From this the solution for x_1 and x_2 is obtained.

QUADRATIC EQUATIONS - YBC 7289 (YBC - YALE BABYLONIAN COLLECTION)

The Babylonians method is procedural (algorithm). Students should be introduced to this procedural approach for its usefulness and for historical reasons. For simplicity and to avoid repetition we change the numbers in the Babylonian text from base 60 to base 10.

BM 13901 (British Museum) - Old Babylonian Collection

Example 1: I have added the area and two thirds of the side of my square and it is $\frac{7}{12}$. (Find side?)

Solution: You take 1

Two thirds of 1, is $\frac{2}{3}$

Half of this $\frac{1}{3}$, you multiply by $\frac{1}{3}$

And the result $\frac{1}{9}$ you add to $\frac{7}{12}$

And the result $\frac{25}{36}$ has $\frac{5}{6}$ as its square root.

$\frac{1}{3}$ which you multiplied by itself, you subtract from $\frac{5}{6}$

And the result $\frac{1}{2}$ *or* 0.50 is the side of the square.

The above procedure solved the quadratic equation: $x^2 + \frac{2}{3}x = \frac{7}{12}$

for 'positive' solutions.

For a full solution we consider at line 5 of the solution procedure, both the positive and negative square root that is both $\frac{5}{6}$ and - $\frac{5}{6}$. Then from the procedure:

$$x_1 = \frac{5}{6} - \frac{1}{3} = \frac{3}{6} = \frac{1}{2}, \quad x_2 = -\frac{5}{6} - \frac{1}{3} = -\frac{7}{6}$$

Example 2: I have added seven times the side of my square to eleven times it's area and it is 6.25.
(Find side?)

SOLUTION: Take 7 and 11
Multiply 11 by 6.25 and it is 68.75
Halve 7 and obtain 3.50
Multiply 3.50 by 3.50 (and obtain 12.25)
Add the result 12.25 to 68.75 (and obtain 81)
And the result 81 has 9 as its square root
Subtract 3.50 which was multiplied by itself from 9 and obtain 5.50
(Divide 5.50 by 11 - but to the Babylonians it is): The reciprocal of 11 does not divide.
What shall I multiply by 11 so that 5.50 results? 0.50 is its factor. 0.50 is the (side of) square.

Here the solution obtained is the 'positive' solution of the quadratic equation :

$$11x^2 + 7x = 6.25$$

The multiplication by 11 (line 2 of solution) has the effect of turning this into a quadratic equation in 11x :

$$(11x)^2 + 7 \times (11x) = 6.25 \times 11 = 68.75$$

where the coefficient of the square term is 1.

For if u is substituted for $11x$, the quadratic equation is transformed to:

$$u^2 + 7u = 68.75$$

The positive solution of the above equation, from line 3 to line 7 of the solution procedure is:

$$u = 5.50$$

Then from the substitution: $u = 11x$

$$11x = 5.50 \quad \textit{and} \quad x = 5.50 \div 11 \quad (\textit{line} \quad 8 \quad \textit{procedure})$$

$$x = 0.50$$

It is clear from the solution procedure that the solutions for a general quadratic equation of form:

$$ax^2 + bx = c$$

$$is: \quad x = \frac{\pm\sqrt{\left(\frac{b}{2}\right)^2 + ac} - \frac{b}{2}}{a}$$

The importance of the Babylonian method is the algorithmic procedure of solution without mathematical symbols besides the fact that the quadratic formula can be inferred from it. This and the fact that essentially the method in use at present is the Babylonian method makes it the ideal way of introducing quadratic equations. Their method will suit those students who are 'put' off more by mathematical symbols than by mathematical reasoning.

BABYLONIAN - PYTHAGORAS THEOREM

Example 1:

The following is a translation of a Babylonian tablet in the British museum:

4 *is the length and* 5 *the diagonal. What is the breadth?*

Its size is not known.

4 *times* 4 *is* 16

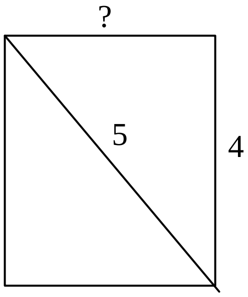
5 *times* 5 *is* 25

You take 16 *from* 25 *and there remains* 9.

What times what shall I take in order to get 9*?*

3 *times* 3 *is* 9

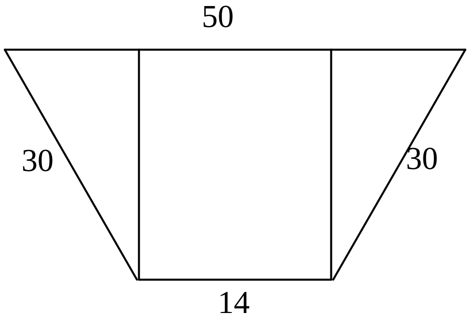
3 *is the breadth.*



Example 2:

A trapezoid 30 is the length, 30 the second length, 50 the upper width, 14 the lower width. (Find the Area)

Solution:



30 times 30 is 900

Subtract 14 from 50 and the remainder is 36

Half of it is 18

18 times 18 is 324

Subtract 324 from 900 and the remainder is 576

What should I multiply by itself so that the result will be 576? 24 times 24 is 576.

24 is the dividing line.

Add 50 and 14 (the widths), and the result is 64.

Half of it is 32

Multiply 24 (the dividing line), by 32, and the result is 768.

Area of trapezoid is 768

That the Babylonians used the 'Pythagoras' theorem is not the issue here. What is of interest is the way they used it. It may be categorised as Babylonian right triangle procedure. The procedural method employed by the Babylonians are important for two main reasons: 1) They are algorithmic and can almost be translated word by word into a programming language as a computer program to solve such problems. They therefore serve as a good introduction to algorithms. 2) This gives a method of teaching such topics without spending too much time on mathematical symbols, and also as indicated earlier will benefit those students who are frightened by mathematical symbols.

Conclusion: The Egyptians method makes it possible to see the difference between arithmetic operations on numbers as utility procedures and as abstracted operations. This is because as utility operations, addition is explicitly used to the procedure for any one of them uses are all 'types of addition'. The above few examples show that a truly inclusive curriculum will improve mathematics education by giving a historical, multicultural and multiracial perspective to mathematics, and by the introduction of varied methods. The hope is that this approach will make the mathematics classroom less alienating for students from marginalized groups and encourage such students to also pursue careers in the mathematical sciences. Because the History of Mathematics is a specialized field of study, there is vast material on the history of mathematics which are easily available in most university libraries and certain sites on the internet. The challenge is to mathematics educators to use this vast material to design a culturally and racially inclusive mathematics curriculum. It is clear that new mathematics textbooks which reflects the content of the inclusive curriculum would need to be written and this is also a challenge to mathematics educators.

Appendix:

Adapting the Egyptian Method of Multiplication to construct a Multiplication Table

x	2	3	4	5	6	7	8	9	10	
1 + 1	**2** 2	**3** 3	**4** 4	**5** 5	**6** 6	**7** 7	**8** 8	**9** 9	**10** 10	
2 + 1	**4** 2	**6** 3	**8** 4	**10** 5	**12** 6	**14** 7	**16** 8	**18** 9	**20** 10	
3 + 1	**6** 2	**9** 3	**1 2** 4	**15** 5	**18** 6	**21** 7	**24** 8	**27** 9	**30** 10	
4 + 1	**8** 2	**12** 3	**16** 4	**20** 5	**24** 6	**28** 7	**32** 8	**36** 9	**40** 10	
5 + 1	**10** 2	**15** 3	**20** 4	**25** 5	**30** 6	**35** 7	**40** 8	**45** 9	**50** 10	
6 + 1	**12** 2	**18** 3	**24** 4	**30** 5	**36** 6	**42** 7	**48** 8	**54** 9	**60** 10	
7 + 1	**14** 2	**21** 3	**28** 4	**35** 5	**42** 6	**49** 7	**56** 8	**63** 9	**70** 10	
8 + 1	**16** 2	**24** 3	**32** 4	**40** 5	**48** 6	**56** 7	**64** 8	**72** 9	**80** 10	
9 + 1	**18** 2	**27** 3	**36** 4	**45** 5	**54** 6	**63** 7	**72** 8	**81** 9	**90** 10	
10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	

From this we can say that the multiplication table is actually the multiple-addition table.

References:

Aaboe, A. (1975) *Episodes from the Early History of Mathematics.* New Mathematical Library, Mathematical Association of America.

Adas, M. (1989) *Machines as the Measure of Men – Science Technology, and Ideologies of Western Dominance* Cornell University Press.

Allen, D. (1997) *www.math.tamu.edu/~don.allen/history/babylon/babylon.htm*

Archibald, R. C. (1941) *Outline of the History of Mathematics (Fifth Edition).* Mathematical Association of America, Inc Oberlin, Ohio, U.S.A.

Arshad, R. (2001) *Institutionalising racial equality in school education*. MCT (Multicultural Teaching) Volume 20 Number 1 pg 21

Ascher, M., Ascher, R. (1997) *Mathematics of the Incas - Code of the Quipu.* Dover Publications Inc. Mineola, New York.

Dei, G. J. S., Mazzuca, J., McIsaac, E., Zine,J. (1997) *Reconstructing 'DROP-OUT'- A critical Ethnography of the Dynamics of Black Students Disengagement from School.* University of Toronto Press, Toronto.

Flinders, D. J. Introduction and Vignette (2001) *Forum: What Should Schools Teach?*

a) Apple, M. W. *The Politics of What is and isn't Taught*.

b) Noddings, N. *How we Teach Matters*.

Journal of Curriculum and Supervision, Volume 16, #2 Winter.

Gillings, R. J. (1982) *Mathematics in the Times of the Pharaohs*. Dover Publications, Inc. New York.

Harding, S. ed. (1993) *The "Racial" Economy of Science: toward a more democratic future*. Bloomington, Indiana University Press.

Newman, J. ed. (2000) *The World of Mathematics (Volume 1).* Dover Publications Inc, New York.

Robins, G., Shute, C. (1987) *The Rhind Mathematical Papyrus - an ancient Egyptian text.* Dover Publications Inc., New York.