Application of Linear Mixed-Effects Models in Longitudinal Data: A Case Study

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ABSTRACT

Longitudinal data are used in the research of growth, development, and change. Such data consist of the same subjects repeatedly measured over time. The longitudinal data typically need some structured covariance models, and the errors often exhibit heteroscedasticity and dependence.

Longitudinal data possess a hierarchical data structure in the sense that the repeated measurements are viewed as a separate level nested within an individual. The main objective of this study was to investigate hierarchical linear models / linear mixed-effects models for longitudinal data. The visual-search data are used to demonstrate the advantages of utilizing the methodology. Several recommendations and advantages on the application of linear mixed-effects models in longitudinal modeling are discussed.

Introduction

Longitudinal data are used in the research on growth, development, and change. Such data consist of measurements on the same subjects repeatedly over time. To describe the pattern of individual growth, make predictions, and gain more insight into the underlying causal relationships related to developmental pattern requires studying the structure of measurements taken on different occasions (Goldstein, 1979). The errors in longitudinal data often exhibit heteroscedasticity and dependence, which call for structured covariance models. Longitudinal data typically possess a hierarchical structure that the repeated measurements are nested within an individual. While the repeated measures are the first level, the individual is the second-level unit and groups of individuals are higher-level units (Hox, 2000). To take heterogeneity and dependence into account, one must include them as parts of the model (Muthen & Satorra, 1989). Longitudinal studies often have unbalanced data structure because subjects withdraw from studies for their personal reasons.

Hierarchical linear models (HLMs) / linear mixed-effects (LMEs) models are an alternative for analyzing longitudinal data. In a HLM varied covariance structures can be imposed on the residuals based on the nature of the data. Thus, hierarchical linear models are well suited for longitudinal data that have variable occasion time, unbalanced data structure, and constrained covariance model for residual errors. Furthermore, if there appear variances in an outcome to be explained by second-level or higher-level variables, hierarchical linear models have advantages over standard multiple regression and other statistical methods because they can analyze multilevel data (Draper, 1995) and incorporate the heteroscedasticity and dependence into the model. For example, hierarchical linear modeling provides a mathematical form that allows researchers to investigate the underlying theory about the functional relationship among the variables in each level (Heck & Thomas, 2000). The variance of an outcome variable is partitioned into "between" and "within" variances, which would increase the precision of estimates.

The main objective of this study was to investigate hierarchical linear models / linear mixed-effects models in modeling multilevel longitudinal data. The literature regarding hierarchical linear models / linear mixed-effects models is first reviewed. The visual-search data of Peterson and Kramer (2001) are reanalyzed via hierarchical linear models/ linear mixed-effects models. Subsequently, many insights and advantages of the application of hierarchical linear models / linear mixed-effects models are summarized and discussed.

Hierarchical Linear Models / Linear Mixed-Effects Models

Hierarchical linear models (HLMs) are indispensable tools for analyzing hierarchically structured data in educational, medical, organizational, geographic, social behavioral, and child growth research (Draper, 1995; Goldstein, 1995; Pollack, 1998). For examples, Francis, Fletcher, Stuebing, Davidson, and Thompson (1991) applied individual growth curves approach to the analysis of change of recovery of cognitive function following pediatric closed head injury. Webster, Mendro, Orsak, and Weerasinghe (1996) discussed related issues of the application of ordinary least squares (OLS) regression and HLM in identifying school and teacher effects. Kreft and Yoon (1994) address the advantages of hierarchical linear models for studying school effectiveness research. Huttenlocher, Haight, Bryk, Seltzer, and Lyons (1991) applied HLM in studying early vocabulary growth. Barnett, Raudenbush, Brennan, and Pleck (1995) utilized HLM in a longitudinal study regarding job change, marital experiences and change in psychological distress. Kidwell, Mossholder, and Bennet (1997) studied the contextual effects on organizational citizenship behavior. In these fields, data are hierarchical in nature, e.g., data are collected by sampling schools, then sampling classes within schools, and then students within classes.

Hierarchical linear models are also known as “multilevel models” (Goldstein, 1995), “random coefficient models,” or “mixed effects” models (Longford, 1993). The term “hierarchical linear model” captures two distinct characteristics of the model. First, the models are appropriate for data that are hierarchically structured with lower-level units nested within higher-level units. Secondly, the parameters in the model can be modeled as having a hierarchical linear structure (Raudenbush, 1993). An intuitive understanding of the term “random coefficient model” or “multilevel model” can be explained from experimental design and the distribution of parameters in regression form. Random coefficient models include the parameters (i.e., intercepts, slopes) that describe the distributions of the variances of group-level statistics (i.e., group intercepts or group slopes). Therefore, they are so-called “random effects” or “random coefficients.” In experimental research, the random effects describe the levels of treatments that are assumed be sampled from a population and inferences refer to this population. The effects in this experimental research are not constant. In fixed-effects design, all possible treatments are included and the inferences are only applied to the specified treatments in the experiment. The effects in fixed-effect design are constant. Random coefficient models can also describe a type of linear or nonlinear forms where the parameters are assumed to vary from a certain probability distribution. In contrast, the coefficients in an ordinary regression are considered fixed and are estimated from the sample data (Heck & Thomas, 2000).

Hierarchical linear models / linear mixed-effects models allow researchers to analyze hierarchically nested data with two or more levels. A two-level hierarchical linear model consists of two submodels: individual-level (level-1) and group-level (level-2). The parameters in a group-level model specify the unknown distribution of individual-level parameters. The intercept and slopes at individual-level can be specified as random. Substituting the level-2 equations for the slopes into the level-1 model yields a linear mixed-effects model. Linear mixed-effects models are mixed-effects models in which both fixed and random effects occur linearly in the model function (Pinheiro & Bates, 2000). Researchers can specify the effects of level-1 coefficients depending on their research interest or the empirical evidence shown in data.

Hierarchical Linear Models for Longitudinal Data

When modeling longitudinal data, the repeated measurements are the level-1 units (i.e., a separate level below individuals). The individual is the second-level unit, and more levels can be added for possible group structures (Hox, 2000).

The basic model at the lowest level, also regarded as repeated-measures level, for the application of hierarchical linear model in longitudinal data can be formulated as:

$$\text{Level-1:} \quad \mathrm{Y}_{ti} = b_{0\mathrm{j}} + b_{1j} c_{ti} + b_{2\mathrm{i}} x_{\mathrm{ti}} + r_{\mathrm{ti}}, \tag{1}$$

where Y_{ti} is the measure for an individual i at time t, c is the time variable indicating the time of measurement for this individual, x_{ti} is the time-varying covariate, and $r_{\mathrm{t}i}$ is the residual error term.

$$\text{Level-2:} \quad b_{0i} = g_{00} + g_{01} W_{\mathrm{i1}} + u_{0\mathrm{i}} \tag{2a}$$

$$b_{1i} = g_{10} + g_{11} W_{\mathrm{i1}} + u_{1\mathrm{i}} \tag{2b}$$

$$b_{2i} = g_{20} + g_{21} W_{\mathrm{i1}} + u_{2\mathrm{i}}\,. \tag{2c}$$

In this level-2 equation, *W* is the time-invariant covariate for this individual. After substituting level-2 equations into level-1, the combined or the linear mixed-effects model is:

$$Y_{ti} = ?_{00} + ?_{10}c_{ti} + ?_{20}x_{ti} + ?_{01}W_{i1} + ?_{11}W_{i1}c_{ti} + ?_{21}x_{ti}W_{i1} + u_{0i} + u_{1i}c_{ti} + u_{2i}x_{ti} + r_{ti}\,. \tag{3}$$

The level-1 model is a within-individuals model and the level-2 model is a between-individuals model (Anderson, 2001). Note that there is no time-invariant covariate in level-2 before introducing the variable W. The variance and covariance of the *u*'s are the variances and covariances of the random intercept and slopes. After introducing the variable *W*, the variance and the covariance of *u*'s are the variance and covariance of residual intercept and slopes after partialing out the variable *W*. More time-invariant variables can be added sequentially into level-2 to get different models. The reduction in variance of *u*'s could provide an estimate of variance in intercepts and slopes accounted for by those *W*'s (MacCallum & Kim, 2000). This linear mixed-effects model does not require that every individual must have the same number of observations because of possible withdrawal from study. The residual errors can be the form of the first-order of autoregression or moving-average process or autoregressive moving-average process (Jones, 1993; Vonesh & Chinchilli, 1997). In some cases, heterogeneous error variances can be employed in the model because the variances in this model are allowed to increase or decrease with time. The assumption of common variance shared by all individuals is removed (Carlin & Louis, 1996; Jones, 1993).

Methods

The visual-search data (Peterson & Kramer, 2001) are reanalyzed by formulating a two-level linear mixed-effects model. There were eighteen subjects (4 males and 14 females) in this study. They are students of the University of Illinois in age from 17 to 31.

The 45 blocks of trials were divided into 6 epochs where one epoch contained the trials from the first half or the second half of the session. The independent variables are onset distractor (present or absent), configuration (repeated or new), and block. The dependent variable is the mean response time and the data were originally analyzed using three-way repeated measures ANOVA with epoch, onset, and configuration as the three factors. The results indicated that:

1. The response was slower when there is an onset distractor present (F (1, 17)=15.38, p<.01).
2. The response to a repeated configuration was faster than to a new configuration (F (1,17)=45.35, p<.01).
3. The response was slower at the earlier epoch (F (5, 85)=45.76, p<.01).
4. There was an interaction of epoch and configuration, with contextual benefits increasing at later epoch. The response was faster for old configurations at the later epoch (F (5, 85)=3.33, p<.05).
5. There is no interaction between epoch and the onset, nor interaction between onset and configuration are needed.

One of the 18 subjects (subject 153) is evident as an outlier (Ker, 2002). This subject was excluded from this study. Cubic linear mixed-effects models were employed to reanalyze the visual-search data. The model formulation followed the standard model building procedures of linear mixed-effects models. The most complex form, which includes all main-effects, all two-way interactions, and three-way interaction terms, is utilized as the preliminary level-1 model. The preliminary level-2 model has intercept as well as linear and quadratic terms of time, and cubic term of time. Detailed demonstrations of model building can be found in the literature (Ker, 2002).

It was found that general positive definite structure is more appropriate. Heteroscedastic and autoregressive model provides better fit to the visual-search data. It was found that there is more variability in new configuration than old configuration (Ker, 2002). One approach to model the heteroscedasticity of the within-subject errors is to use different variance for each level of configuration. The standard error for the new configuration is about 31% higher than for old configuration. The relative model-fit information of the cubic LME is given in Table 1.

Plots of the population prediction (fixed), within-group predictions (Subject), and the observed values for each subject on the cubic LME fits are shown in Figure 1. Population predictions are obtained by setting random- effects to zero whereas within-group predictions use estimated random effects. These two prediction lines follow the observed values closely indicating the cubic LME model provide good explanation to the data.

Results

The fixed-effects structures of these two models contain significant treatment effects for configuration, distractor presence, time, and the interaction of configuration and time. Effect coding was used for the treatment effects where the coding for new configuration is -1 and 1 for old configuration, and the coding for distractor present is -1 and 1 for distractor absent. The negative parameter estimate for configuration indicates that subjects responds to old configuration more quickly than to new configuration. The parameter estimate for time is negative indicating that higher mean RTs tend to occur early in the experiment and decrease over time. The negative parameter estimate for distractor indicates that subjects are slower when distractor was present than it is absent. With respect to the interaction of configuration and time, old configuration decrease at a faster rate over time than new configuration. These conclusions are consistent with the literature by Peterson and Kramer (2001).

The advantages of utilizing linear mixed-effects models are that LMEs can formulate the precise forms of the model to fit, is flexible in specifying the covariance models, can model the heteroscedasticity and dependence exist in residual errors.

Table 1 is the model information for the cubic LME. For the old configuration, distractor absent, and at time 10, the fitted value is 885.71 (1137.29-41.27-9.21-25.72*10+0.70*10^2 -0.01*10^3-0.39*10). For the new configuration, distractor absent and at time 10, the fitted value is 976.05 (1137.29+41.27-9.21-25.72*10+0.70*10^2-0.01*10^3+0.39*10). Holding other variables constant, the difference between the new configuration and old configurtion at time 10 is 90.34. The difference between distractor present and distractor absent can be realized by the same way. Furthermore, the standard error for the new configuration is about 31% higher than for old configuration. There exists dependency in within-subject errors. The estimated single correlation parameter for the AR(1) model is 0.12.

Discussions and Conclusions

Longitudinal data have a multilevel structure in that the occasions of measurements are nested within individuals (MacCallum & Kim, 2000). Longitudinal data cannot be adequately explained by a simple fixed model; however, the development of hierarchical linear models, allowing structured covariance matrices and the parameters in a regression model to vary over individuals, provides an alternative for analyzing longitudinal data. Hierarchical linear models (HLMs) assume that the pattern of individual growth has the same functional form for all individuals, but individuals may have different parameters (i.e, they grow in different ways). The individual differences are represented by the variations in intercepts and slopes (MacCallum & Kim, 2000). Time-varying or other individual attributes can be added to the model as covariates.

Thus, each individual has his or her own growth curve specified by the regression coefficients depending on individual's characteristics to help explaining individual differences (Hox, 2000).

Since linear mixed-effects models make predictions curves for population and individuals, it can provide the information regarding the performance of individual student relative the average performance. It is especially helpful in education if teachers can identify those extraordinary students, e.g., those students whose individual growth curves are highly above or below the population curves. Special programs or remedies can be given based on students' abilities. Moreover, linear mixed-effects models provide explicit forms for individual growth curves. Each student can have his or her individual growth curve, specified by the regression coefficients that may depend on his or her own characteristics. Those curves can describe the changes of growth over time for individual differences. The significances of predictors may be of interests for further investigation on students' growth.

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Table 1
The Relative Model-Fit Information for the Cubic LME Model

Random Effects

	Intercept	Time	Time^2	Residual
Standard Deviation	206.89	7.68	0.12	90.19

Fixed Effects

Parameter	Value	Std.Error	DF	t-value	p-value
Intercept	1137.29	50.82	3037	22.38	<.0001
Conf	-41.27	3.59	3037	-11.48	<.0001
Dist	-9.21	1.76	3037	-5.23	<.0001
Time	-25.72	2.39	3037	-10.72	<.0001
Time^2	0.70	0.09	3037	8.16	<.0001
Time^3	-0.01	0.001	3037	-5.64	<.0001
Conf*Time	-0.39	0.14	3037	-2.78	0.0054

Note. (a) Model fit: AIC=37190.36, BIC=37286.78, logLik=-18579.18. (b) Correlation structure: AR(1); parameter estimate(s): Phi=0.12. (c) Variance function structure: different standard deviations per stratum; parameter estimates: New/Old=1.31/1.

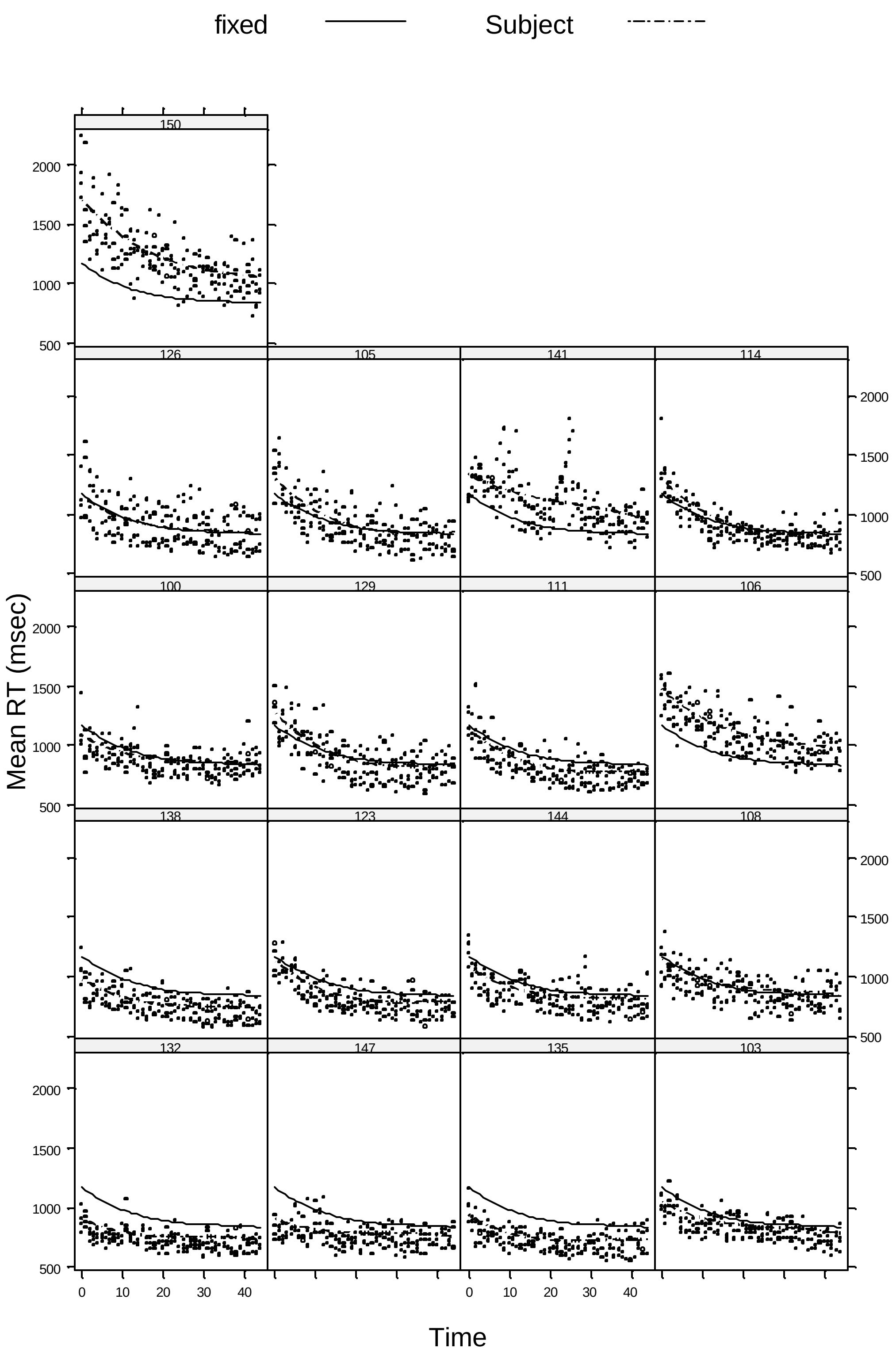

Figure 1. Population predictions (fixed), within-group predictions (Subject), and observed values for the cubic LME.