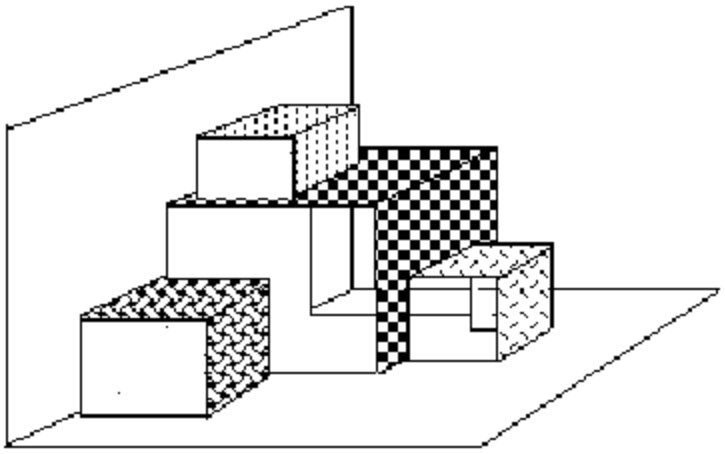
POP-UP ENGINEERING

and

GEOMETRIC CONCEPTS



MATHEMATICS WORKSHOP

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CONTENTS

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Abstract:
Hands on Technology
POP-UP Engineering and GEOMETRIC concepts.

The main purpose of this workshop is to focus upon a complementary approach to the study of, and the investigation into, concepts related to the teaching of Geometry at the primary level.

Participants in this workshop will use the mechanism of paper engineering (pop-up mechanism) to produce models 3-D geometric models which illustrate geometric concepts.

The process of the construction of these models allows for **building imagery, testing predictions, arousing and satisfying curiosity.**

It is envisaged that this approach to the teaching of geometric concepts will provide grounds for discussion, enrichment, clarification of and ownership of ideas, cross curriculum integration and exploration. It has the potential to reduce the apparent difficulty students experience with the study of geometric concepts.

This approach transports the students beyond the static two-dimensional aspects of geometry to a more dynamic, life like geometric world where shapes 'move' and may be seen as part of a three dimensional scene.

In this workshop we will use pop-up engineering mechanisms to produce geometric and non geometric figures in an attempt to evoke the imagery of three dimensional figures, and to consider how the approach might be effectively used in our classroom.

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MATHEMATICS WORKSHOP

POP-UP ENGINEERING and GEOMETRIC CONCEPTS

1. GENERAL INFORMATION

Introduction:-

The main purpose of this workshop is to focus upon a complementary approach to the study of, and the investigation into, teaching of Geometrical concepts at Primary level. Children and adults have maintained a fascination with pop-up books and cards. It is, I imagine, the thrill of experiencing the unexpected objects popping up as the books and/or cards are opened, and seeing two dimensional shapes transformed into three dimensional figures. This too may have contributed to the children's curiosity. It is this intrigue which has led me to explore the possibility of utilising the mechanism of paper engineering (as it is called) to construct geometric figures, and thus introduce children to geometric concepts. With a very limited knowledge of paper engineering I found that it continued to hold my interest, arouse my curiosity, and to challenge me as I tried to master the techniques 'to get it to work'. But as I progressed, and once the technique of cutting, folding, scoring and interpreting the drawings were mastered, it was my imagination and my use of visual perception and imagery which allowed me to continue to create and to build some very fascinating models.
I am of the opinion that this type of visual experience, complemented with intrigue and problem solving will enhance the children's thinking and understanding in the study of geometry.

The focus of the workshop is to use paper engineering mechanism to construct geometric figures. To explore the potential of this approach to create interest, arouse curiosity, and motivate students in such a way that they will acquire a better understanding of geometric concepts taught at primary level.
I hope that this approach will reduce the apparent difficulty students experience with the study of geometry. What is very evident with this approach is that it removes the static two-dimensional aspect of geometry and brings a more life like attribute to geometric shapes, that is the shapes move and can be actually seen as part of a three dimensional scene.

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Historically, since the 18th century, some amusing children's books contained, flaps to turn, peep-holes, and cut-outs. In 1855, Dean of London published the first pop-up book called *Little Red Riding Hood* (Hiner, 1985). Today we see many children's books and cards which are produced with a pop-up mechanism. The mechanism used produces a whole range of exciting possibilities and movements such as translation, rotation, reflection, enlargement etc. Hiner (1985) suggested that 'by choosing the right mechanism and then adapting and developing it to suit your particular need, you can express an idea, a mood far more vividly than any static picture ever can.'

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Skills Required:

To create the 'working' pop-up shapes the following skills are necessary.

1). Cutting,

2). Scoring,

3). Folding and

4). Gluing

1. CUTTING

Materials:

- a sharp pointed craft knife
- a metal edge ruler
- heavy card board or Artline Mat.

Cutting the paper;

Place your index and thumb fingers on the ruler.

Position the tip of the knife at the edge of the metal ruler to make at about 10 degrees. Then with a slight pressure on the tip of the knife cut the paper.

This strategy allows you to see your work and minimises the risk of the knife 'running away' from the expected line of cut.

At times a pair of scissors may be preferred.

Heavy cardboard, Artline Mat or any other appropriate material should be placed under your paper to protect the surface of the table.

(Pine board or other similar soft material are not recommended for this purpose).

Note:

Lines to be cut in the diagrams will be indicated as solid lines.

2. SCORING

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Material:-

- **a pair of scissors or an empty ball point pen.**

Scoring

Scoring refers to making an impression on the paper in such a way that a 'sharp' line is formed.

To produce this effect use the tip of a scissors blade or an empty ball point pen. These lines become **folds.**

Scoring produces a well defined line and/or a hinge. It is one of the important mechanisms in paper engineering. It adds to the quality of the hinge, and it gives a professional look to the work when completed.

3. FOLDING

Material:-

- a ruler or the handle of a toothbrush

Folding

Fold along the scored line and drag the edge of a ruler or the handle of a toothbrush or other similar object along the fold to produce a 'sharp' edge.

Note:

Lines to be folded in the diagrams will be indicated as **dotted lines.**

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4. GLUING

Materials:-

- a quick setting jar of glue,
- a ruler, and
- a piece of cloth, approximately 15 cm X 15 cm.

Gluing

Apply a small quantity of glue where required.

Too much glue will not only result in a messy job, but will take extra time to dry.

Use a clean edge ruler and gently push the glue away and towards the edge of the paper.

In this way the glue will spread evenly and any trapped air will be pushed out.

Remove any extra glue immediately, which may have leaked through when spreading.

Materials and Tools

Choosing the appropriate quality paper will enhance the model produced. The recommended quality paper is manilla 100 - 120 g/m^2.

This leaflet was typed on 80 g/m^2 paper (A4 size). This is regarded to be too thin and will not usually suffice. Manila paper of 100 - 120 g/m^2 is recommended especially if the fibres in the paper could make a good crease when it is scored and folded.
Very thick paper usually is not ideal for pop-ups. The size you choose for your pop-ups is rather important. An A4 size paper folded in half along the shorter length is usually convenient for the students to use with these activities.
The following materials/tools are recommended for the activities.

- a pencil,
- A4 paper
- manilla cardboard,
- a pair of scissors

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- a ruler(metal edge)
- a craft knife
- a pair of compasses,
- a cutting board or heavy card board
- a tube of glue,
- a protractor.
- 1 cm graph paper,
- coloured A4 size paper

2. Pertinent Topics in Mathematics which may emerge from the Activities

The following list suggests some of the mathematics topics and processes which may emerge from the various activities.

- Accurate construction of geometrical figures
- Angles, parallel lines and planes
- Classification of 2D shapes and 3D figures
- Polygons and their properties
- Perimeter, area, volume.
- Transformation:- translation, reflection, rotation, enlargement and reduction of shapes/figures
- Making predictions and justifying these orally
- Two and three-dimensional representation of objects.
- The language of Mathematics

3. WORKSHOP ACTIVITIES

Introduction:

Each activity is subdivided into a number of stages (5) which you should follow through to fully benefit from the experience.

Stage 1: Instructs you **to fold and cut** the paper along specific guide lines.

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Stage 2: Invites you **to imagine** a particular process associated with the first stage and asks you to answer related questions.

Stage 3. Invites you **to predict** a result if a set of instructions were followed.

Stage 4. Invites you **to carry out** the instructions and to test your imagery and predictions.

Stage 5. Invites you **to discuss** your out comes, respond to the questions asked and compare your 'model' with that of your colleagues.

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ACTIVITY 1(a)

Stage 1. Fold and Cut

- Fold an A4 size paper in half by bringing together the shorter ends. Crease the fold
- Construct/Draw a square on the folded paper such that a side of the square lies on the fold.
  (please see Figure 1 below)
- Cut the sides of the square which are perpendicular to the fold.
- Score along the dotted line.

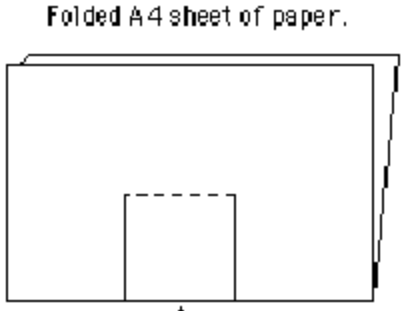

Figure 1

- Opening the folded paper only slightly, push the folded edge of the square through to the inside, until the scored line prevents further movement.
  Crease along the scored line.

Stage 2 Imagine

What figure will emerge when the folded paper is slowly opened?

Stage 3 Predict

What figure will result when the angle of the fold is 90°?

Stage 4 Proceed

Proceed to carry out the actions described above and test your imagery and prediction.
Were you surprised at your creation?
Did the creation coincide with your imagery and prediction?
Slowly close the paper and describe the transformation of the figure.

Stage 5 Discussion

Where would you place this activity in a regular mathematics classroom?
Name the mathematical concepts which could be accompanied with this model.
Suggest a title for this activity.
Any other?.

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ACTIVITY 1(b)

In this activity you are asked to use your acquired skill and produce a model which shows three cubes adjacent to each other and of different sizes (please see figure 3 below).
You should sketch a diagram before proceeding with the construction. Also follow the stages as in activity 1(a) above.

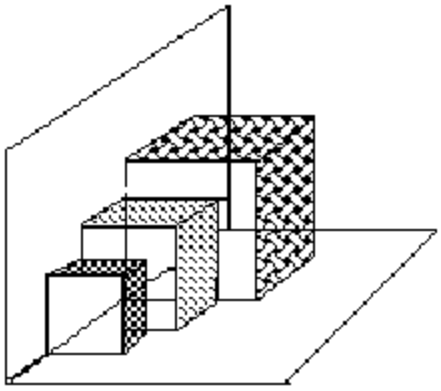

Figure 2

ACTIVITY 1(c)

In this activity using your acquired skill produce a model which shows three cubes one infront of each other, in the formation of a stair and the cubes increase in size (please see figure 3 below)

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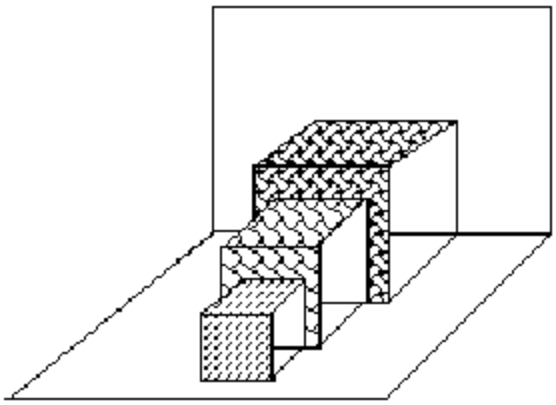

Figure 3

ACTIVITY 1 (d)

In this activity you are asked, with out the aid of a cue, to construct a cube with in a cube with in a cube. In other words three cubes of different sizes, one within the other.

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ACTIVITY 2

Stage 1 Fold and Cut

Fold an A4 size sheet of paper as in Activity 1.

- Draw one of the rectangles as shown below in Figure 4
- Cut, crease and fold as in Activity 1.

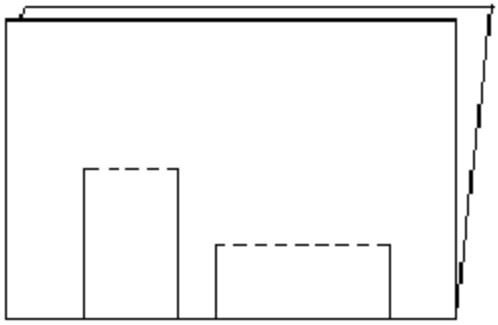

Figure 4

Stage 2 Imagine

What figure will emerge when the folded paper is slowly opened.?

Imagine What figure will emerge if you had chosen to construct the other rectangle in Fig. 4. ?

Describe how these two figures are different.

Stage 3 Predict

What figure will result when the angle of the fold is 90°?

Stage 4 Proceed

Proceed and carry out the actions as in Activity 1 and test your imagery and prediction.

How could the activities above be used in the classroom to explore mathematical content ?

Stage 5 Discussion:

As above.

Suggest a title for this activity.

ACTIVITY 3

Stage 1 Fold and Cut

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- Repeat activity 1 stage 1.
- Open the sheet of paper at 180 degrees and look at how the 3D figure has transformed into a 2D shape.
- Describe this shape.

Stage 2 Imagine

- Imagine that the A4 size sheet of paper was not folded and that it was placed on the table.
- Draw on this sheet of paper the appropriate lines such that when the paper is folded and cut the pop-up figure will result in a cube.

Stage 3 Predict

Check your predictions

Stage 4

- Did you notice that in activities 1 - 3 the open ends of the created figure is a square?
- Explore the possibility of changing these open ends to rectangles.

Stage 5 Discussion

Discuss your possibility with your colleagues and test your diagrams.

ACTIVITY 4

Use the information from activity 3 above and construct a podium showing three platforms of different heights
(please see figure 5)

Proceed by using the stages suggested in activity 1

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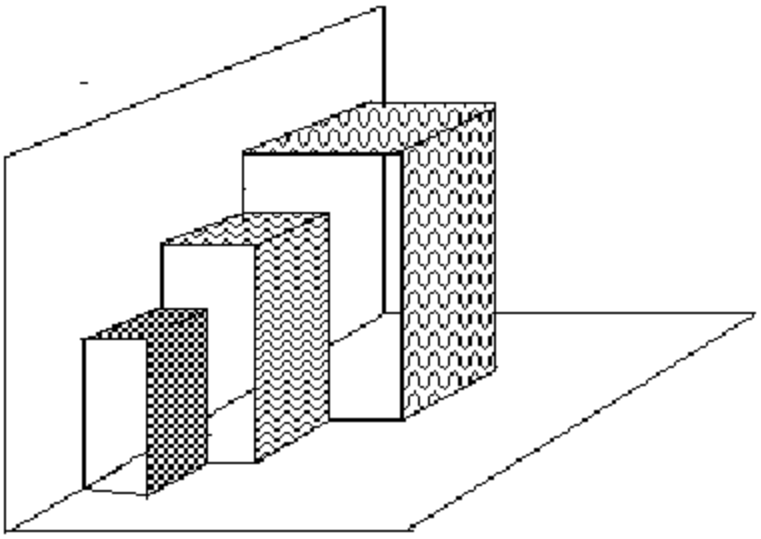

Figure 5

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ACTIVITY 5

Stage 1 Fold and cut

Fold the paper and draw the parallelogram as shown in Figure. 6
Score, cut and fold as in Activity 1.

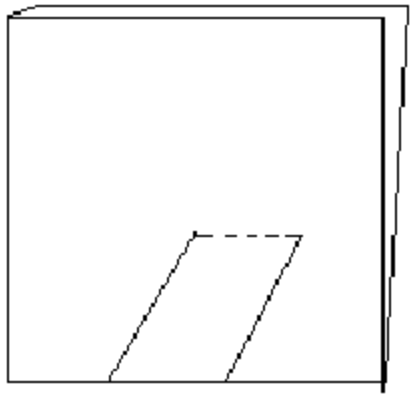

Figure 6

Stage 2 Imagine

What figure will emerge when the folded paper is slowly opened?

Stage 3 Predict

What figure will result when the angle of the fold is 90°?

Stage 4 Proceed

- Proceed to carry out the actions as in Activity 1 and test your imagery and prediction.
- Does the resultant figure resemble your prediction?
- Look at the open 'faces' of the figure. What shape(s) would be needed to enclose the figure fully?
- Draw the net of this figure.

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